REMARKS

This is a response to the Final Office Action mailed June 3, 2004 in relation to the above-identified patent application.

Item 1. (Page 2):

Applicant's arguments in March 2004 were not persuasive because the recitation of interleaver had not been given patentable weight as the recitation occurred in the preamble.

Independent claims 6 and 14 have been amended by citing specific structural limitations for the interleaver such as "... providing two interim output ~~components~~ optical beams, which are generally orthogonal to each other in polarization direction, for one input optical beam at the interleaver input" and "... are selected from the following table such that two final output optical beams, wherein each final output optical beam consists of a group of optical components of interleaved optical wavelengths, are obtained at the interleaver output for the input optical beam". These structural limitations for the interleaver can be found in paragraphs [0003]-[0009], [0092] and [0096] of the specification.

Please Note: in the amended claims, the "~~single strikethrough~~" indicates the deleted portion in applicant's current response and the "~~double strikethrough~~" indicates the deleted portion in applicant's last response (mailed on March 9, 2004).

It is respectfully requested that the amended independent claims 6, 14, dependent claims 12, 13, and 15 are allowable.

Item 2 (Page 2) and Item 3 (Page 3):

Rejection of Claims 6 and 14 under 35 U.S.C. 102(b) as being anticipated by Cheng et al (5,768,005).

It is respectfully submitted that the instant application is fundamentally different from Cheng's (5,768,005) in both structure and functionality.

Structure Difference:

In Cheng's ('005), birefringent element (12) is orientated to separate an optical input beam into two optical beams (see Fig. 1c, from Zin_F to $Z12_F$, col.3 lines 27-31). Birefringent element (18) is orientated to first combine two optical beams into one optical beam (from $Z16_F$

to $Z18_F$ in Fig. 1c, col. 3 lines 36-38)). After the beam being reflected, birefringent element (18) separates one optical beam into two optical beams (see from $Z18_F$ to $Z18_{R \text{ after reflection}}$ in Fig.1c, col.3 lines 47-49). And then, birefringent element (12) combines two optical beams into one optical beam (from $Z14_R$ to $Z12_R$ in Fig. 1c, col.3 lines 51-53). In summary, birefringent elements (12) and (18) are orientated to provide beam separation and combination in Cheng's ('005).

In the instant application, the birefringent elements of the birefringent element assembly are orientated to provide birefringent phase delay and there is no optical beam separation and combination in these birefringent elements. In Fig.9 of the instant application, elements (23), (24), and (25) are the birefringent elements of the birefringent element assembly. When optical beams pass through (23), (24), and (25), there is no optical beam separation/combination as shown in Fig.9 and Fig.10 for the optical beams propagating from location 3 to location 7. As shown in Fig.10, there are two optical beams at location 3 (each beam occupies a beam position box as shown at 3) and the same two optical beams remain at the same beam positions at location 7 (each beam occupies the same beam position box as shown at 7).

It is respectfully submitted that when a birefringent element is orientated to provide birefringent phase delay to an optical beam, the optical beam's propagation direction must be along one principal dielectric axis and perpendicular to the other two principal dielectric axes of a birefringent element (or crystal) – in such a case, no optical beam separation/combination can be obtained. In comparison, when a birefringent element (or crystal) is used to separate/combine optical beams, the optical beam's propagation directions must not be along and perpendicular to any principal dielectric axis of the birefringent element. This is a sharp difference between the instant application and Cheng's ('005).

To support this key point, two technical references are respectfully submitted along with this response. Reference 1, adopted from the book "Optical Electronics", discusses details on how the birefringent phase delay (called as "phase retardation" in the book) is obtained in a birefringent element (called as "birefringent plate" in the book). On page 11, the "principal dielectric axes" were defined along (x, y, z) directions. The subsequent materials in Reference 1 showed that birefringent retardation (phase delay) is obtained when an optical beam propagates along one principal dielectric axis and perpendicular to the other two principal dielectric axes of a birefringent element and there is no optical beam separation/combination.

Reference 2 is a product description on optical beam displacers made of birefringent elements (from www.thorlabs.com). As shown in Reference 2, the optical beam's propagation direction must be at an angle (not 0 or 90 degree) with respect to the optic axis in order to displace/separate an optical beam. Optic axis is one of the principal dielectric axes as shown in Fig. 1-1 on page 14 of Reference 1. If the optical beam's propagation direction is along one principal dielectric axis (and thus perpendicular to the other two principal axes), no optical beam displacement/separation can be obtained – instead, birefringent phase delay is obtained.

Functionality Difference:

The structure difference described above results in functionality difference between the instant application and Cheng's ('005).

Cheng's ('005) discussed optical isolating devices (i.e., optical isolator). When an optical beam passes through the optical isolating device from its input to its output, the optical isolating device is to prevent reflection light propagate backward from the output to the input. The devices disclosed in Cheng's ('005) provide one output optical beam for one input optical beam – see Zin_F (one input optical beam) and $Z12_R$ (one output optical beam) in Fig.1c. For two output optical beams, two input optical beams must be provided as shown in Fig.7 in Cheng's ('005).

In comparison, the interleaver of the instant application is to separate an input optical beam into two output optical beams of interleaved optical wavelengths. <u>An interleaver provides two output optical beams for one input optical beam</u> – see Figs. 9 and 10, position <u>0</u> (one input optical beam) and position <u>17</u> (two output beams). The two output optical beams cannot be seen from Fig. 9 because they overlap with each other from the view of Fig. 9. However, Fig. 10 and paragraph [0096] clearly indicates that there are two output optical beams at the interleaver output (two boxes occupied at position <u>17</u>).

Amended Claims:

To differentiate the instant application from Cheng's ('005), independent claims 6 and 14 have been amended to include "...wherein <u>the birefringent element assembly comprises three birefringent elements which are orientated to provide birefringent phase delays to the optical beams passing through them</u>" for claim 6 and "wherein the birefringent element assembly comprises two birefringent elements which <u>are orientated to provide birefringent phase delays to the optical beams passing through them</u>" for claim 14. The amended claims clearly indicated that

the birefringent elements were constructed to provide birefringent phase delay instead of optical beam displacement in the instant application – a sharp difference from Cheng's ('005).

It is respectfully requested that the amended independent claims 6, 14, dependent claims 12, 13, and 15 are allowable.

Item 4 (Page 3):

Rejection of Claims 6, 12 and 14 under 35 U.S.C. 102(b) as being anticipated by Cheng et al (5,471,340).

It is respectful to submit that the instant application is fundamentally different from Cheng's (5,471,340) in both structure and functionality.

Structure Difference:

In Cheng's ('340), birefringent element (24) is orientated to separate an optical input beam into two optical beams (see col.3 lines 42-51 and Fig.1C: optical beam LF in box 24 is separated into two optical beams LF1 in box 26 and LF2 in box 28 in the "FORWARD" direction). Similarly, birefringent element (24A) is orientated to separate an optical beam into two optical beams (see col.5 lines 44-48 and Fig.3C: optical beam LF in box 24 is separated into two optical beams LF1 and LF2 in box 24B). Birefringent element (24B) is orientated to displace the optical beams further apart and to move LF2 beneath LF1 in their physical locations (see col.5 lines 48-52 and Fig.3C: LF2 is moved beneath LF1 in box 30). Further, birefringent element (32A) is orientated to displace the optical beam LF1's position to the right (see col.5 lines 58-61 and Fig.3C: LF1 is above LF2 in box 32A and LF1 is moved to the right as shown in box 34). In summary, birefringent elements (24, 24A, 24B, 32A) and the like in Cheng's ('340) are orientated to displace (move) the position of an optical beam according to the polarization direction of the optical beam. In some cases, this results in optical beams' separation/combination.

In the instant application, the birefringent elements of the birefringent element assembly are orientated to provide birefringent phase delay and there is on optical beam displacement /separation/combination in these birefringent elements. In Fig.9 of the instant application, elements (23), (24), and (25) are the birefringent elements of the birefringent element assembly. When optical beams pass through (23), (24), and (25), there is no optical beam displacement/separation/combination as shown in Fig.9 and Fig.10 for the optical beams

propagating from location 3 to location 7. As shown in Fig.10, there are two optical beams at location 3 (each beam occupies a beam position box as shown at 3) and the same two optical beams remain at the same beam positions at location 7 (each beam occupies the same beam position box as shown at 7).

It is respectfully submitted that when a birefringent element is orientated to provide birefringent phase delay to an optical beam, the optical beam's propagation direction must be along one principal dielectric axis and perpendicular to the other two principal dielectric axes of a birefringent element (or crystal) – in such a case, no optical beam separation/combination can be obtained. In comparison, when a birefringent element (or crystal) is used to displace/separate/combine optical beams, the optical beam's propagation directions must not be along and perpendicular to any principal dielectric axis of the birefringent element. This is a sharp difference between the instant application and Cheng's ('340).

Functionality Difference:

The structure difference described above results in functionality difference between the instant application and Cheng's ('340).

Cheng's ('340) discussed optical isolating devices (i.e., optical isolator). The devices disclosed in Cheng's ('340) can only provide one output optical beam for one input optical beam. As shown in the top portion "1 TO 2" of Fig.1C, one input optical beam LF (shown in the top box 24 in the "FORWARD" direction) results in one output optical beam LF (shown in the bottom box 24 in the "REVERSE" direction). Similarly, the bottom portion "2 TO 3" of Fig.1C shows that one optical input beam (2 in the "FORWARD" direction) results in only one optical output beam (3 in the "REVERSE" direction).

In comparison, the interleaver of the instant application is to separate an input optical beam into two output optical beams of interleaved optical wavelengths. An interleaver provides two output optical beams for one input optical beam – see Figs. 9 and 10, position 0 (one input optical beam) and position 17 (two output beams). The two output optical beams cannot be seen from Fig. 9 because they overlap with each other from the view of Fig. 9. However, Fig. 10 and paragraph [0096] clearly indicates that there are two output optical beams at the interleaver output (two boxes occupied at position 17).

Amended Claims:

To differentiate the instant application from Cheng's ('340), independent claims 6 and 14 have been amended to include "...wherein the birefringent element assembly comprises three birefringent elements which are orientated to provide birefringent phase delays to the optical beams passing through them" for claim 6 and "wherein the birefringent element assembly comprises two birefringent elements which are orientated to provide birefringent phase delays to the optical beams passing through them" for claim 14. The amended claims clearly indicated that the birefringent elements were constructed to provide birefringent phase delay instead of optical beam displacement in the instant application – a sharp difference from Cheng's ('340).

It is respectfully requested that the amended independent claims 6, 14, dependent claims 12, 13, and 15 are allowable.

Item 5 (Page 4):

Claims 13 and 15 are objected to as being dependent upon a rejected base claim.

It is respectfully requested that dependent claims 13 and 15 are allowable if the amended independent claims 6 and 14 are allowable.

Correction to Typos in Table III for Claim 6 (Previously submitted on March 9, 2004):

In the patent publication US 2003/0025998 A1 (Feb. 6, 2003), there are four typos for Table III under claim 6 and paragraph [0143]:

"100_1" (happened at two places) should be "φ_1" and "100_3"(happened at two places) should be "φ_3", respectively.

Table III in the specification paragraph [0129] is correct. Please help to make correction if this application is allowed to issue.

****

Please note that applicant has a new address. Please address all correspondence to

Bin Zhao

14 Figaro

Irvine, CA 92606

Respectfully submitted,

By: ______________________

Bin Zhao

Telephone: (949) 266-6800

Fax: (801) 858-8132

Email: zhaob2001@yahoo.com

The Holt, Rinehart and Winston
Series in Electrical Engineering
published by Saunders College Publishing

M. E. Van Valkenburg, Senior Consulting Editor
Adel S. Sedra, Series Editor/Electrical Engineering
Michael R. Lightner, Series Editor/Computer Engineering

ALLEN AND HOLBERG
CMOS Analog Circuit Design
BELANGER, ADLER AND RUMIN
Introduction to Circuits with Electronics: An Integrated Approach
BOBROW
Elementary Linear Circuit Analysis, 2/e
BOBROW
Fundamentals of Electrical Engineering
CHEN
Linear System Theory and Design
CHEN
System and Signal Analysis
COMER
Digital Logic and State Machine Design, 2/e
COMER
Microprocessor-Based System Design
COOPER AND McGILLEM
Probabilistic Methods of Signal and System Analysis, 2/e
GHAUSI
Electronic Devices and Circuits: Discrete and Integrated
HOSTETTER, SAVANT AND STEFANI
Design of Feedback Control Systems, 2/e
HOUTS
Signal Analysis in Linear Systems
JONES
Introduction to Optical Fiber Communication Systems
KARNI AND BYATT
Mathematical Methods in Continuous and Discrete Systems
KENNEDY
Operational Amplifier Circuits: Theory and Application
KUO
Digital Control Systems
LASTMAN AND SINHA
Microcomputer-Based Numerical Methods for Science and Engineering
LATHI
Modern Digital and Analog Communication Systems, 2/e
LEVENTHAL
Microcomputer Experimentation with the IBM PC
LEVENTHAL
Microcomputer Experimentation with the Intel SDK-86
LEVENTHAL
Microcomputer Experimentation with the Motorola MC6800 ECB
MCGILLEM AND COOPER
Continuous and Discrete Signal and System Analysis, 3/e
NAVON
Semiconductor Microdevices and Materials
PAPOULIS
Circuits and Systems: A Modern Approach
RAMSHAW AND VAN HEESWIJK
Energy Conversion: Electric Motors and Generators
SADIKU
Elements of Electromagnetics
SCHWARZ
Electromagnetics for Engineers
SCHWARZ AND OLDHAM
Electrical Engineering: An Introduction
SEDRA AND SMITH
Microelectronic Circuits, 3/e
SINHA
Control Systems
VAN VALKENBURG
Analog Filter Design
VRANESIC AND ZAKY
Microcomputer Structures
WARNER AND GRUNG
Semiconductor-Device Electronics
WASSER AND FLYNN
Introduction to Arithmetic for Digital Systems Designers
WOLOVICH
Basic Analysis and Design
YARIV
Optical Electronics, 4/e

Reference 1

Optical Electronics

Fourth Edition

Amnon Yariv
California Institute of Technology

HRW SAUNDERS
Saunders College Publishing
A Division of Holt, Rinehart and Winston, Inc.
Philadelphia Chicago Fort Worth San Francisco
Montreal Toronto London Sydney Tokyo

Text Typeface: Times Roman
Compositor: General Graphic Services
Senior Acquisitions Editor: Barbara Gingery
Managing Editor: Carol Field
Project Editor: Martha Brown
Manager of Art and Design: Carol Bleistine
Art Director: Christine Schueler
Art and Design Coordinator: Doris Bruey
Text Designer: Rita Naughton
Cover Designer: Lawrence R. Didona
New Text Artwork: Vantage Art
Director of EDP: Tim Frelick
Production Manager: Bob Butler
Marketing Manager: Denise Watrobsky

Printed in the United States of America

OPTICAL ELECTRONICS, 4e

0-03-047444-2

Library of Congress Catalog Card Number: 90-050825

1234 039 987654321

THIS BOOK IS PRINTED ON **ACID-FREE, RECYCLED** PAPER

Preface

The five years that have intervened since the appearance of the third edit of *OPTICAL ELECTRONICS* witnessed significant technical developme in the field and the emergence of some major trends. A few of the import developments are

1. Optical fiber communication has established itself as the key commu cation technology.
2. The semiconductor laser and especially the longer wavelength GaInA InP version has emerged as the main light source for high-data-rate opti fiber communication systems.
3. Quantum well semiconductor lasers started replacing their conventio counterparts for high-data-rate long distance communication and m other sophisticated applications including ultra-low threshold and mo locked lasers.
4. Optical fiber amplifiers are causing a minor revolution in fiber comr nication due to their impact on very long distance transmission and large scale optical distribution systems.

The accumulated weight of the new developments was such that wl I last taught the course at Caltech in 1989 I found myself using a substan fraction of course material that was not included in the text. The fou edition brings this material into the fold. The main additions to the th edition, include major revisions and new chapters dealing with

1. Jones calculus and its extension to Faraday effect elements.
2. Radiometry and infrared detection.

density. We start by considering the second and fourth terms on the right of (1.2-11)

$$\frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}\mathbf{e}\cdot\mathbf{e}\right) + \mathbf{e}\cdot\frac{\partial \mathbf{p}}{\partial t}$$

Using the relations

$$\mathbf{p} = \varepsilon_0 \chi_e \mathbf{e}$$
$$\varepsilon = \varepsilon_0(1+\chi_e) \tag{1.3-20}$$

we obtain

$$\frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}\mathbf{e}\cdot\mathbf{e}\right) + \mathbf{e}\cdot\frac{\partial \mathbf{p}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\varepsilon}{2}\mathbf{e}\cdot\mathbf{e}\right) \tag{1.3-21}$$

Since we assumed the medium to be lossless, the last term must represent the rate of change of electric energy density stored in the vacuum as well as in the electric dipoles; that is,

$$\frac{\mathscr{E}_{\text{electric}}}{\text{Volume}} = \frac{\varepsilon}{2}\mathbf{e}\cdot\mathbf{e} \tag{1.3-22}$$

The magnetic energy density is derived in a similar fashion using the relations

$$\mathbf{m} = \chi_m \mathbf{h}$$
$$\mu = \mu_0(1+\chi_m)$$

resulting in

$$\frac{\mathscr{E}_{\text{magnetic}}}{\text{Volume}} = \frac{\mu}{2}\mathbf{h}\cdot\mathbf{h} \tag{1.3-23}$$

Considering only the positive traveling wave in (1.3-17), we obtain from (1.3-22) and (1.3-23)

$$\begin{aligned}\frac{\overline{\mathscr{E}_{\text{magnetic}}} + \overline{\mathscr{E}_{\text{electric}}}}{\text{Volume}} &= \left(\frac{\varepsilon}{2}\right)\overline{(e_x^+)^2} + \left(\frac{\mu}{2}\right)\overline{(h_y^+)^2}\\ &= \frac{\varepsilon}{4}|E_x^+|^2 + \frac{\mu}{4}|H_y^+|^2\\ &= \frac{\varepsilon}{4}|E_x^+|^2 + \frac{\mu}{4}\frac{|E_x^+|^2}{\eta^2}\\ &= \tfrac{1}{2}\varepsilon|E_x^+|^2\end{aligned} \tag{1.3-24}$$

where the second equality is based on (1.1-12), and the third and fourth use (1.3-15). Comparing (1.3-24) to (1.3-19), we get

$$\frac{I}{\overline{\mathscr{E}}/\text{Volume}} = \frac{1}{\sqrt{\mu\varepsilon}} = c \tag{1.3-25}$$

where $\overline{\mathscr{E}} = \overline{\mathscr{E}_{\text{magnetic}}} + \overline{\mathscr{E}_{\text{electric}}}$ is the electromagnetic field energy and c is the phase velocity of light in the medium. In terms of the electric field we get

$$I = \frac{c\varepsilon|E|^2}{2} \tag{1.3-26}$$

1.4 WAVE PROPAGATION IN CRYSTALS—THE INDEX ELLIPSOID

In the discussion of electromagnetic wave propagation up to this point, we have assumed that the medium was isotropic. This causes the induced polarization to be parallel to the electric field and to be related to it by a (scalar) factor that is independent of the direction along which the field is applied. This situation does not apply in the case of dielectric crystals. Since the crystal is made up of a regular periodic array of atoms (or ions), we may expect that the induced polarization will depend in its magnitude and direction, on the direction of the applied field. Instead of the simple relation (1.3-20) linking **p** and **e**, we have

$$\begin{aligned}P_x &= \varepsilon_0(\chi_{11}E_x + \chi_{12}E_y + \chi_{13}E_z)\\ P_y &= \varepsilon_0(\chi_{21}E_x + \chi_{22}E_y + \chi_{23}E_z)\\ P_z &= \varepsilon_0(\chi_{31}E_x + \chi_{32}E_y + \chi_{33}E_z)\end{aligned} \tag{1.4-1}$$

where the capital letters denote the complex amplitudes of the corresponding time-harmonic quantities. The 3×3 array of the χ_{ij} coefficients is called the electric susceptibility tensor. The magnitude of the χ_{ij} coefficients depends, of course, on the choice of the x, y, and z axes relative to that of the crystal structure. It is always possible to choose x, y, and z in such a way that the off-diagonal elements vanish, leaving

$$\begin{aligned}P_x &= \varepsilon_0\chi_{11}E_x\\ P_y &= \varepsilon_0\chi_{22}E_y\\ P_z &= \varepsilon_0\chi_{33}E_z\end{aligned} \tag{1.4-2}$$

These directions are called the *principal dielectric axes of the crystal*. In this book we will use only the principal coordinate system. We can, instead of using (1.4-2), describe the dielectric response of the crystal by means of the electric permeability tensor ε_{ij}, defined by

$$\begin{aligned}D_x &= \varepsilon_{11}E_x\\ D_y &= \varepsilon_{22}E_y\\ D_z &= \varepsilon_{33}E_z.\end{aligned} \tag{1.4-3}$$

From (1.4-2) and the relation

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

we have

$$\begin{aligned}\varepsilon_{11} &= \varepsilon_0(1 + \chi_{11})\\ \varepsilon_{22} &= \varepsilon_0(1 + \chi_{22})\\ \varepsilon_{33} &= \varepsilon_0(1 + \chi_{33})\end{aligned} \tag{1.4-4}$$

Birefringence

One of the most important consequences of the dielectric anisotropy of crystals is the phenomenon of birefringence in which the phase velocity of an optical beam propagating in the crystal depends on the direction of polarization of its **e** vector. Before treating this problem mathematically, we may pause and ponder its physical origin. In an isotropic medium the induced polarization is independent of the field direction so that $\chi_{11} = \chi_{22} = \chi_{33}$, and, using (1.4-4), $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \varepsilon$. Since $c = (\mu\varepsilon)^{-1/2}$, the phase velocity is independent of the direction of polarization. In an anisotropic medium the situation is different. Consider, for example, a wave propagating along z. If its electric field is parallel to x, it will induce, according to (1.4-2), only P_x and will consequently "see" an electric permeability ε_{11}. Its phase velocity will thus be $c_x = (\mu\varepsilon_{11})^{-1/2}$. If, on the other hand, the wave is polarized parallel to y, it will propagate with a phase velocity $c_y = (\mu\varepsilon_{22})^{-1/2}$.

Birefringence has some interesting consequences. Consider, as an example, a wave propagating along the crystal z direction and having at some plane, say $z = 0$, a linearly polarized field with equal components along x and y. Since $k_x \neq k_y$, as the wave propagates into the crystal the x and y components get out of phase and the wave becomes elliptically polarized. This phenomenon is discussed in detail in Section 9.2 and forms the basis of the electrooptic modulation of light.

Returning to the example of a wave propagating along the crystal z direction, let us assume, as in Section 1.3, that the only nonvanishing field components are e_x and h_y. Maxwell's curl equations (1.3-5) and (1.3-4) reduce, in a self-consistent manner, to

$$\begin{aligned}\frac{\partial e_x}{\partial z} &= -\mu \frac{\partial h_y}{\partial t}\\ \frac{\partial h_y}{\partial z} &= -\varepsilon_{11} \frac{\partial e_x}{\partial t}\end{aligned} \tag{1.4-5}$$

Taking the derivative of the first of Equations (1.4-5) with respect to z and then substituting the second equation for $\partial h_y/\partial z$ gives

$$\frac{\partial^2 e_x}{\partial z^2} = \mu\varepsilon_{11} \frac{\partial^2 e_x}{\partial t^2} \tag{1.4-6}$$

If we postulate, as in (1.3-10), a solution in the form

$$e_x = E_x e^{i(\omega t - k_x z)} \tag{1.4-7}$$

then Equation (1.4-6) becomes

$$k_x^2 E_x = \omega^2 \mu\varepsilon_{11} E_x$$

Therefore, the propagation constant of a wave polarized along x and traveling along z is

$$k_x = \omega\sqrt{\mu\varepsilon_{11}} \tag{1.4-8}$$

Repeating the derivation but with a wave polarized along the y axis, instead of the x axis, yields $k_y = \omega\sqrt{\mu\varepsilon_{22}}$.

Index Ellipsoid

As shown above, in a crystal the phase velocity of a wave propagating along a given direction depends on the direction of its polarization. For propagation along z, as an example, we found that Maxwell's equations admitted two solutions: one with its linear polarization along x and the second along y. If we consider the propagation along some arbitrary direction in the crystal, the problem becomes more difficult. We have to determine the directions of polarization of the two allowed waves, as well as their phase velocities. This is done most conveniently using the so-called index ellipsoid

$$\frac{x^2}{\varepsilon_{11}/\varepsilon_0} + \frac{y^2}{\varepsilon_{22}/\varepsilon_0} + \frac{z^2}{\varepsilon_{33}/\varepsilon_0} = 1 \tag{1.4-9}$$

This is the equation of a generalized ellipsoid with major axes parallel to x, y, and z whose respective lengths are $2\sqrt{\varepsilon_{11}/\varepsilon_0}$, $2\sqrt{\varepsilon_{22}/\varepsilon_0}$, and $2\sqrt{\varepsilon_{33}/\varepsilon_0}$. The procedure for finding the polarization directions and the corresponding phase velocities for a *given* direction of propagation is as follows: Determine the ellipse formed by the intersection of a plane through the origin and normal to the direction of propagation and the index ellipsoid (1.4-9). The directions of the major and minor axes of this ellipse are those of the two allowed polarizations,[5] and the lengths of these axes are $2n_1$ and $2n_2$, where n_1 and n_2 are the indices of refraction of the two allowed solutions. The two waves propagate, thus, with phase velocities c_0/n_1 and c_0/n_2, respectively, where $c_0 = (\mu_0\varepsilon_0)^{-1/2}$ is the phase velocity in vacuum. A formal proof of this procedure is given in References [2–4].

To illustrate the use of the index ellipsoid, consider the case of a uniaxial crystal (that is, a crystal with a single axis of threefold, fourfold, or sixfold symmetry). Taking the direction of this axis as z, symmetry considerations dictate that $\varepsilon_{11} = \varepsilon_{22}$.[6] Defining the principal indices of refraction n_o and n_e

[5]These are actually the directions of the **D**, not of the **E**, vector. In a crystal these two are separated, in general, by a small angle; see References [2] and [3].

[6]See, for example, J. F. Nye, *Physical Properties of Crystals*. New York: Oxford University Press, 1957.

by

$$n_o^2 \equiv \frac{\varepsilon_{11}}{\varepsilon_0} = \frac{\varepsilon_{22}}{\varepsilon_0} \qquad n_e^2 \equiv \frac{\varepsilon_{33}}{\varepsilon_0} \tag{1.4-10}$$

the equation of the index ellipsoid (1.4-9) becomes

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \tag{1.4-11}$$

This is an ellipsoid of revolution with the circular symmetry axis parallel to z. The z major axis of the ellipsoid is of length $2n_e$, whereas that of the x and y axes is $2n_o$. The procedure of using the index ellipsoid is illustrated by Figure 1-1.

The direction of propagation is along s and is at an angle θ to the (optic) z axis. Because of the circular symmetry of (1.4-11) about z, we can choose, without any loss of generality, the y axis to coincide with the projection of s on the x-y plane. The intersection ellipse of the plane normal to s with the ellipsoid is shaded in the figure. The two allowed polarization directions are parallel to the axes of the ellipse and thus correspond to the line segments OA and OB. They are consequently perpendicular to s as well as to each other. The two waves polarized along these directions have, respectively,

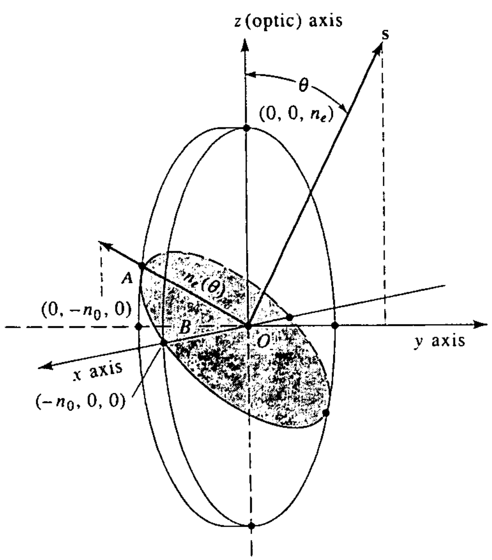


Figure 1-1 Construction for finding indices of refraction and allowed polarization for a given direction of propagation s. The figure shown is for a uniaxial crystal with $n_x = n_y = n_o$.

indices of refraction given by $n_e(\theta) = |OA|$ and $n_o = |OB|$. The first of these two waves, which is polarized along OA, is called the *extraordinary wave*. Its direction of polarization varies with θ following the intersection point A. Its index of refraction is given by the length of OA. It can be determined using Figure 1-2, which shows the intersection of the index ellipsoid with the y-z plane.

Using the relations

$$n_e^2(\theta) = z^2 + y^2$$

$$\frac{z}{n_e(\theta)} = \sin\theta$$

and the equation of the ellipse

$$\frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

we obtain

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2} \tag{1.4-12}$$

Thus, for $\theta = 0°$, $n_e(0°) = n_o$, and for $\theta = 90°$, $n_e(90°) = n_e$.

The ordinary wave remains, according to Figure 1-1, polarized along the same direction OB independent of θ. It has an index of refraction n_o. The amount of birefringence $n_e(\theta) - n_o$ thus varies from zero for $\theta = 0°$ (that is, propagation along the optic axis) to $n_e - n_o$ for $\theta = 90°$.

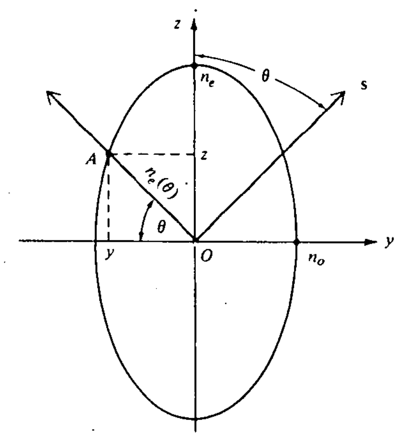


Figure 1-2 Intersection of the index ellipsoid with the z-y plane. $|OA| = n_e(\theta)$ is the index of refraction of the extraordinary wave propagating in the direction s.

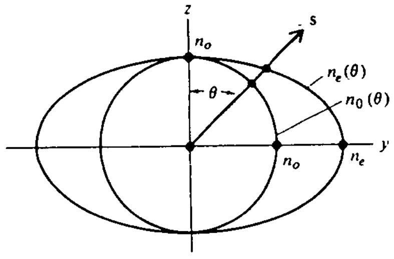


Figure 1-3 Intersection of s-z plane with normal surfaces of a positive uniaxial crystal ($n_e > n_o$).

Normal (index) Surfaces

Consider the surface in which the distance of a given point from the origin is equal to the index of refraction of a wave propagating along this direction. This surface, not to be confused with the index ellipsoid, is called the normal surface. It is constructed using the index ellipsoid (Figure 1-1). The normal surface of the extraordinary ray is constructed by measuring along each direction $s(\theta, \phi)$ the corresponding index $n_e(\theta, \phi)$, which is the distance OA in Figure 1-1. For a uniaxial crystal, this results in an ellipsoid of revolution about the z axis as illustrated by the outer line in Figure 1-3. For the ordinary ray we plot the distance $OB = n_0$ (which is independent of θ, ϕ), resulting in the inner sphere of Figure 1-3.

1.5 JONES CALCULUS AND ITS APPLICATION TO PROPAGATION IN OPTICAL SYSTEMS WITH BIREFRINGENT CRYSTALS

Many sophisticated optical systems, such as electrooptic modulators (to be discussed in Chapter 9) involve the passage of light through a train of polarizers and birefringent (retardation) plates. The effect of each individual element, either polarizer or retardation plate, on the polarization state of the transmitted light can be described by simple means. However, when an optical system consists of many such elements, each oriented at a different azimuthal angle, the calculation of the overall transmission becomes complicated and is greatly facilitated by a systematic approach. The Jones calculus, invented in 1940 by R. C. Jones [5], is a powerful matrix method in which the state of polarization is represented by a two-component vector, while each optical element is represented by a 2 × 2 matrix. The overall transfer matrix for the whole system is obtained by multiplying all the individual element matrices, and the polarization state of the transmitted light is computed by multiplying the vector representing the input beam by the overall matrix. We will first develop the mathematical formulation of the Jones matrix method and then apply it to some cases of practical interest.

We have shown in the previous section that a unidirectional light propagation in a birefringent crystal generally consists of a linear superposition of two orthogonally polarized waves—the eigenwaves. These eigenwaves, for a given direction of propagation, have well-defined phase velocities and directions of polarization. The birefringent crystals may be either uniaxial ($n_x = n_y, n_z$) or biaxial ($n_x \neq n_y \neq n_z$). However, the most commonly used materials, such as calcite and quartz, are uniaxial. In a uniaxial crystal, these eigenwaves are the so-called *ordinary* and *extraordinary* waves, whose properties were derived in Section 1.4. The directions of polarization for these eigenwaves are mutually orthogonal and are called the *slow* and *fast* axes of the crystal for the given direction of propagation. Retardation plates are usually cut in such a way that the c axis lies in the plane of the plate surfaces. Thus the propagation direction of normally incident light is perpendicular to the c axis.

Retardation plates (also called wave plates) are polarization-state converters, or transformers. The polarization state of a light beam can be converted to any other polarization state by using a suitable retardation plate. In formulating the Jones matrix method, we assume that there is no reflection of light from either surface of the plate and the light is totally transmitted through the plate surfaces. In practice, there is some reflection, though most retardation plates are coated to reduce the surface reflection loss. Referring to Figure 1.4, we consider a light beam that is incident normally on a re-

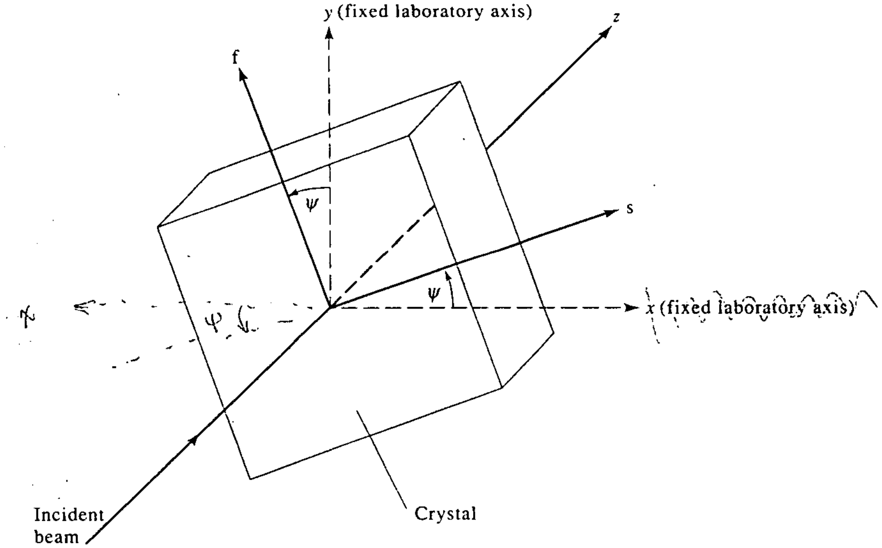


Figure 1-4 A retardation plate rotated at an angle ψ about the z axis. f("fast") and s("slow") are the two principal dielectric axes of the crystal for light propagating along z (see Section 1.4). The x and y axes are fixed in the laboratory frame.

tardation plate along the z axis with a polarization state described by the Jones column vector

$$\mathbf{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{1.5-1}$$

where V_x and V_y are two complex numbers representing the complex field amplitudes along x and y. The x, y and z axes are *fixed* laboratory axes. To determine how the light propagates in the retardation plate, we need to resolve it into a linear combination of the fast and slow eigenwaves of the crystal. This is done by the coordinate transformation

$$\begin{pmatrix} V_s \\ V_f \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{1.5-2}$$

V_s is the slow component of the polarization vector $\mathbf{V}$, whereas V_f is the fast component. The slow and fast axes are fixed in the crystal. The angle between the fast axis and the y direction is ψ. These two components are eigenwaves of the retardation plate and will propagate with their own phase velocities and polarizations as discussed in Section 1.4. Because of the difference in phase velocity, the two components undergo a different phase delay in passage through the crystal. This retardation changes the polarization state of the emerging beam.

Let n_s and n_f be the refractive indices of the slow and fast eigenwaves, respectively. The polarization state of the emerging beam in the crystal coordinate system is thus given by

$$\begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = \begin{pmatrix} \exp\left(-in_s \frac{\omega}{c} l\right) & 0 \\ 0 & \exp\left(-in_f \frac{\omega}{c} l\right) \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix} \tag{1.5-3}$$

where l is the thickness of the plate and ω is the radian frequency of the light beam. The phase retardation is defined as the difference of the phase delays (exponents) in (1.5-3)

$$\Gamma = (n_s - n_f) \frac{\omega l}{c} \tag{1.5-4}$$

Notice that the phase retardation Γ is a measure of the relative change in phase, not the absolute change. The birefringence of a typical crystal retardation plate is small, that is, $|n_s - n_f| << n_s, n_f$. Consequently, the absolute change in phase caused by the plate may be hundreds of times greater than the phase retardation. Let ϕ be the mean absolute phase change

$$\phi = \tfrac{1}{2}(n_s + n_f) \frac{\omega l}{c} \tag{1.5-5}$$

Then Equation (1.5-3) can be written in terms of ϕ and Γ as

$$\begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix} \tag{1.5-6}$$

The Jones vector of the polarization state of the emerging beam in the xy coordinate system is given by transforming back from the crystal to the laboratory coordinate system

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} V_s' \\ V_f' \end{pmatrix} \tag{1.5-7}$$

By combining Equations (1.5-2), (1.5-6), and (1.5-7), we can write the transformation due to the retardation plate as

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = R(-\psi)\, W_0 R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{1.5-8}$$

where $R(\psi)$ is the rotation matrix of (1.5-2) and W_0 is the Jones matrix of (1.5-6) for the retardation plate. These are given, respectively, by

$$R(\psi) = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \tag{1.5-9}$$

and

$$W_0 = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \tag{1.5-10}$$

The phase factor $e^{-i\phi}$ can usually be left out.[7] A retardation plate, characterized by its phase retardation Γ and its azimuth angle ψ, is represented by the product of three matrices

$$\begin{aligned} W &= R(-\psi) W_0 R(\psi) \\ &= \begin{vmatrix} e^{-i(\Gamma/2)}\cos^2\psi + e^{i(\Gamma/2)}\sin^2\psi & -i\sin\frac{\Gamma}{2}\sin(2\psi) \\ -i\sin\frac{\Gamma}{2}\sin(2\psi) & e^{-i(\Gamma/2)}\sin^2\psi + e^{i(\Gamma/2)}\cos^2\psi \end{vmatrix} \end{aligned} \tag{1.5-11}$$

Note that the Jones matrix of a wave plate is a unitary matrix, that is,

$$W^\dagger W = 1$$

where the dagger $\dagger$ signifies the Hermitian conjugate ($W_{ij}^* = (W^\dagger)_{ji}$). The passage of a polarized light beam through a wave plate is described mathematically as a unitary transformation. Many physical properties are invar-

[7] The overall phase factor $\exp(-i\phi)$ is only important when the output field $\mathbf{V}'$ is combined coherently with another field.

iant under unitary transformations; these include the orthogonal relation between the Jones vectors and the magnitude of the Jones vectors. Thus, if the polarization states of two beams are mutually orthogonal, they will remain orthogonal after passing through an arbitrary wave plate.

The Jones matrix of an ideal, homogeneous, linear, thin platelet polarizer oriented with its transmission axis parallel to the laboratory x axis is

$$P_0 = e^{-i\phi}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \tag{1.5-12}$$

where ϕ is the absolute phase accumulated due to the finite optical thickness of the polarizer. The Jones matrix of a polarizer rotated by an angle ψ about z is given by

$$P = R(-\psi)P_0R(\psi) \tag{1.5-13}$$

Thus, if we neglect the (in this case unimportant) absolute phase ϕ, the Jones matrix representations of the polarizers oriented so as to transmit light with electric field vectors parallel to the x and y laboratory axes, respectively, are given by

$$P_x = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \quad \text{and} \quad P_y = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \tag{1.5-14}$$

To find the effect of an arbitrary train of retardation plates and polarizers on the polarization state of polarized light, we multiply the Jones vector of the incident beam by the ordered product of the matrices of the various elements.

Example: A Half-Wave Retardation Plate

A half-wave plate has a phase retardation of $\Gamma = \pi$. According to Equation (1.5-4), an x-cut[8] (or y-cut) uniaxial crystal will act as a half-wave plate, provided the thickness is $t = \lambda/2(n_e - n_0)$. We will determine the effect of a half-wave plate on the polarization state of a transmitted light beam. The azimuth angle of the wave plate is taken as 45° and the incident beam as vertically (y) polarized. The Jones vector for the incident beam can be written as

$$\mathbf{V} = \begin{pmatrix}0\\ 1\end{pmatrix} \tag{1.5-15}$$

[8] A crystal plate is called *x-cut* if its facets are perpendicular to the principal x axis.

and the Jones matrix for the half-wave plate is obtained by using Equation (1.5-11) with $\Gamma = \pi$, $\psi = \pi/4$

$$W = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix} = \begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix} \tag{1.5-16}$$

The Jones vector for the emerging beam is obtained by multiplying Equations (1.5-16) and (1.5-15); the result is

$$\mathbf{V}' = \begin{pmatrix}-i\\ 0\end{pmatrix} = -i\begin{pmatrix}1\\ 0\end{pmatrix} \tag{1.5-17}$$

which corresponds to horizontally (x) polarized light. The effect of the half-wave plate is thus to rotate the input polarization by 90°. It can be shown that for a general azimuth angle ψ, the half-wave plate will rotate the polarization by an angle 2ψ (see Problem 1.7a). In other words, linearly polarized light remains linearly polarized, except that the plane of polarization is rotated by an angle of 2ψ.

When the incident light is circularly polarized, a half-wave plate will convert right-hand circularly polarized light into left-hand circularly polarized light and vice versa, regardless of the azimuth angle. The proof is left as an exercise (see Problem 1.7). Figure 1.5 illustrates the effect of a half-wave plate.

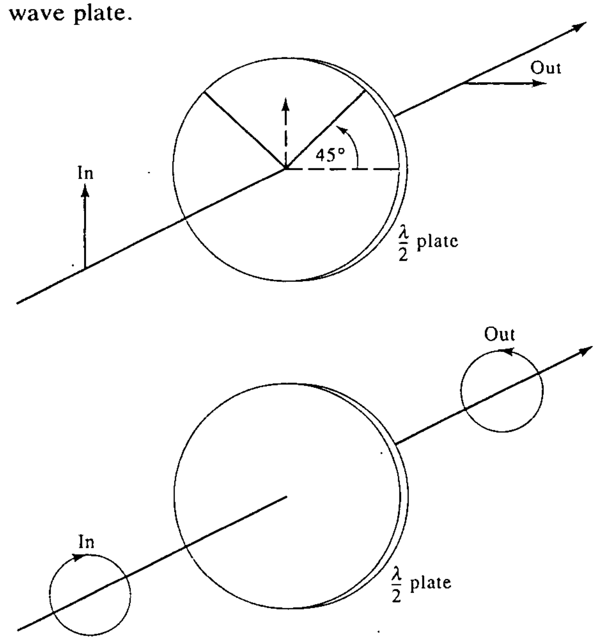


Figure 1-5 The effect of a half-wave plate on the polarization state of a beam.

Example: A Quarter-Wave Plate

A quarter-wave plate has a phase retardation of $\Gamma = \pi/2$. If the plate is made of an x-cut (or y-cut) uniaxially anistropic crystal, the thickness is $t = \lambda/4\ (n_e - n_0)$ (or odd multiples thereof). Suppose again that the azimuth angle of the plate is $\psi = 45°$ and the incident beam is vertically polarized. The Jones vector for the incident beam is given by Equation (1.5-15). The Jones matrix for this quarter-wave plate is

$$W = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \tag{1.5-18}$$

The Jones vector of the emerging beam is obtained by multiplying Equations (1.5-18) and (1.5-15) and is given by

$$\mathbf{V}' = -\frac{i}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \tag{1.5-19}$$

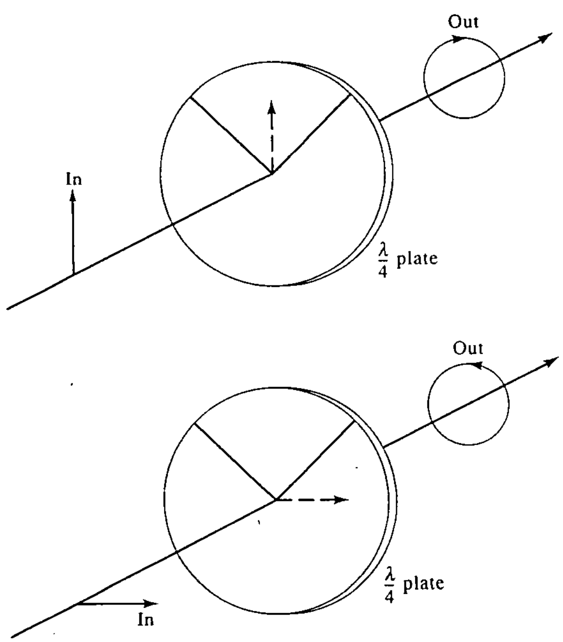


Figure 1-6 The effect of a quarter-wave plate on the polarization state of a linearly polarized input wave.

This is a left-hand circularly polarized light. The effect of a 45°-oriented quarter-wave plate is thus to convert vertically polarized light into left-hand circularly polarized light. If the incident beam is horizontally polarized, the emerging beam will be right-hand circularly polarized. The effect of this quarter-wave plate is illustrated in Figure 1-6.

Intensity Transmission

Up to this point our development of the Jones calculus was concerned with the polarization state of the light beam. In many cases, we need to determine the transmitted intensity. The combination of retardation plates and polarizers is often used to control or modulate the transmitted optical intensity. Because the phase retardation of each wave plate is wavelength-dependent, the polarization state of the emerging beam and its intensity (when polarizers are present) depend on the wavelength of the light. Let us represent the field as a Jones vector

$$\mathbf{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{1.5-20}$$

The intensity is taken using (1.1-12) and (1.3-24) as proportional to:

$$I = \mathbf{V} \cdot \mathbf{V}^* = |V_x|^2 + |V_y|^2 \tag{1.5-21}$$

If the output beam is given by

$$\mathbf{V}' = \begin{pmatrix} V'_x \\ V'_y \end{pmatrix} \tag{1.5-22}$$

the transmissivity of the optical system is calculated as

$$\frac{|V'_x|^2 + |V'_y|^2}{|V_x|^2 + |V_y|^2} \tag{1.5-23}$$

Example: A Birefringent Plate Sandwiched between Parallel Polarizers

Referring to Figure 1-7, we consider a birefringent plate sandwiched between a pair of parallel polarizers. The plate is oriented so that the slow and fast axes are at 45° with respect to the polarizer. Let the birefringence be $n_e - n_0$ and the plate thickness be d. The phase retardation is then given by

$$\Gamma = 2\pi(n_e - n_0)\frac{d}{\lambda} \tag{1.5-24}$$

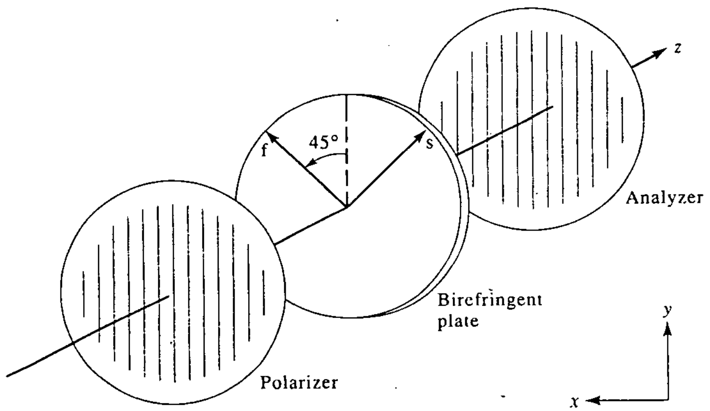


Figure 1-7 A birefringent plate sandwiched between a pair of parallel polarizers.

and the corresponding Jones matrix is, according to Equation (1.5-11), with $\psi = 45°$

$$W = \begin{pmatrix} \cos\frac{1}{2}\Gamma & -i\sin\frac{1}{2}\Gamma \\ -i\sin\frac{1}{2}\Gamma & \cos\frac{1}{2}\Gamma \end{pmatrix} \tag{1.5-25}$$

The incident beam, after it passes through the front polarizer, is polarized parallel to y and can be represented by

$$\mathbf{V} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.5-26}$$

we shall take, arbitrarily, the intensity corresponding to (1.5-26) as unity. The Jones vector representation of the electric field vector of the transmitted beam is obtained as follows:

$$\mathbf{V}' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{1}{2}\Gamma & -i\sin\frac{1}{2}\Gamma \\ -i\sin\frac{1}{2}\Gamma & \cos\frac{1}{2}\Gamma \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \cos\frac{1}{2}\Gamma \end{pmatrix} \tag{1.5-27}$$

The transmitted beam is y polarized with an intensity given by

$$I = \cos^2\tfrac{1}{2}\Gamma = \cos^2\left[\frac{\pi(n_e - n_0)d}{\lambda}\right] \tag{1.5-28}$$

It can be seen from Equation (1.5-27) that the transmitted intensity is a sinusoidal function of the wave number (λ^{-1}) and peaks at $\lambda = (n_e - n_0)d$, $(n_e - n_0)d/2$, $(n_e - n_0)d/3, \ldots$. The wave-number separation between transmission maxima increases with decreasing plate thickness.

Example: A Birefringent Plate Sandwiched between a Pair of Crossed Polarizers

If we rotate the analyzer shown in Figure 1-7 by 90°, then the input and output polarizers are crossed. The transmitted beam for this case is obtained as follows:

$$\mathbf{V}' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{1}{2}\Gamma & -i\sin\frac{1}{2}\Gamma \\ -i\sin\frac{1}{2}\Gamma & \cos\frac{1}{2}\Gamma \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -i \begin{pmatrix} \sin\frac{\Gamma}{2} \\ 0 \end{pmatrix} \tag{1.5-29}$$

The transmitted beam is horizontally (x) polarized with an intensity relative to the input value given by

$$\frac{I_{out}}{I_{in}} = \sin^2\tfrac{1}{2}\Gamma = \sin^2\left[\frac{\pi(n_e - n_0)\,d}{\lambda}\right] \tag{1.5-30}$$

This is again a sinusoidal function of the wave number. The transmission spectrum consists of a series of maxima at $\lambda = 2(n_e - n_0)d$, $2(n_e - n_0)d/3, \ldots$. These wavelengths correspond to phase retardations of $\pi, 3\pi, 5\pi, \ldots$, that is, when the wave plate becomes a "half-wave" plate or odd integral multiples of a half-wave plate.

Circular Polarization Representation

Up to this point we represented the state of the propagating field as a vector $\mathbf{V}$ [Equation (1.5-1)] with components V_x and V_y.

$$\mathbf{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \tag{1.5-31}$$

The orthogonal unit vectors (*basis* vector set) in this representation are

$$\mathbf{V}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbf{V}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.5-32}$$

The above choice is most convenient when dealing with birefringent crystals, since the propagating eigenmodes in this case are linearly and orthogonally polarized. It is often more convenient to express the field in terms of "basis" vectors that are circularly polarized [6]. This is the case, for example, when we propagate through a magnetic medium. We define a wave of unit amplitude seen rotating in the CCW sense by an observer gazing along the $+z$ axis as $\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$, while $\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$ denotes a CW rotating wave. As in the case of the

linearly polarized basis vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, $\begin{Bmatrix}1\\0\end{Bmatrix}$ and $\begin{Bmatrix}0\\1\end{Bmatrix}$ constitute a complete set that can be used to describe a transverse field of arbitrary polarization. Let **V** be some such field. We can write

$$\mathbf{V} = V_x\begin{pmatrix}1\\0\end{pmatrix} + V_y\begin{pmatrix}0\\1\end{pmatrix} \equiv \begin{pmatrix}V_x\\V_y\end{pmatrix} \tag{1.5-33}$$

or alternatively

$$\mathbf{V} = V_+\begin{Bmatrix}1\\0\end{Bmatrix} + V_-\begin{Bmatrix}0\\1\end{Bmatrix} \equiv \begin{Bmatrix}V_+\\V_-\end{Bmatrix} \tag{1.5-34}$$

The $\begin{pmatrix}V_x\\V_y\end{pmatrix}$ and $\begin{Bmatrix}V_+\\V_-\end{Bmatrix}$ representations of a given vector can be derived from each other by a 2×2 matrix[9]

$$\begin{Bmatrix}V_+\\V_-\end{Bmatrix} = \frac{1}{2}\begin{vmatrix}1 & i\\1 & -i\end{vmatrix}\begin{pmatrix}V_x\\V_y\end{pmatrix} \equiv T\begin{pmatrix}V_x\\V_y\end{pmatrix} \tag{1.5-35}$$

$$\begin{pmatrix}V_x\\V_y\end{pmatrix} = \begin{vmatrix}1 & 1\\-i & i\end{vmatrix}\begin{Bmatrix}V_+\\V_-\end{Bmatrix} \equiv S\begin{Bmatrix}V_+\\V_-\end{Bmatrix} \tag{1.5-36}$$

so that $T = S^{-1}$. As an example, consider a (unit) field polarized along x. Its rectangular representation is $\begin{pmatrix}1\\0\end{pmatrix}$, while its rotating representation is

$$\begin{Bmatrix}V_+\\V_-\end{Bmatrix} = \begin{vmatrix}1 & i\\1 & -i\end{vmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{Bmatrix}1\\1\end{Bmatrix} \tag{1.5-37}$$

i.e., equal and in-phase admixture of the two counter-rotating eigenmodes. Conversely, a clockwise, circularly polarized unit wave $\begin{Bmatrix}0\\1\end{Bmatrix}$, for example, is expressed in the rectangular representation by

$$\begin{pmatrix}V_x\\V_y\end{pmatrix} = \begin{vmatrix}1 & 1\\-i & i\end{vmatrix}\begin{Bmatrix}0\\1\end{Bmatrix} = \begin{pmatrix}1\\i\end{pmatrix} \tag{1.5-38}$$

Faraday Rotation In certain optical materials containing magnetic atoms or ions, the natural modes of propagation are the two counter-rotating, circularly-polarized (CP) waves described above. The z direction is usually that of an applied magnetic field or that of the spontaneous magnetization. As in the case of a birefringent crystal, the two CP modes propagate with different phase velocities or, equivalently, have different indices of refraction. This difference is due to the fact that the individual atomic magnetic moments precess in a unique sense about the z axis and thus interact differently (have slightly displaced resonances) with the two CP waves. Using the notation of (1.5-34), we can describe the propagation of a wave with arbitrary transverse polarization by first resolving it, at $z = 0$, into its components $\begin{Bmatrix}V_+(0)\\0\end{Bmatrix}$ and $\begin{Bmatrix}0\\V_-(0)\end{Bmatrix}$ and propagating each component with its appropriate phase delay through the magnetic medium

$$\begin{Bmatrix}V_+(z)\\V_-(z)\end{Bmatrix} = \begin{Bmatrix}V_+(0)\\0\end{Bmatrix}e^{-i(\omega/c)n_+z} + \begin{Bmatrix}0\\V_-(0)\end{Bmatrix}e^{-i(\omega/c)n_-z}$$

$$= e^{-(i/2)(\theta_+ + \theta_-)}\begin{vmatrix}e^{(i/2)(\theta_- - \theta_+)} & 0\\0 & e^{-(i/2)(\theta_- - \theta_+)}\end{vmatrix}\begin{Bmatrix}V_+(0)\\V_-(0)\end{Bmatrix} \tag{1.5-39}$$

where $\theta_\pm \equiv (\omega/c)\, n_\pm z$ is the phase delay for the (+) or (−) circularly polarized wave. Ignoring the prefactor $\exp[-(i/2)(\theta_+ + \theta_-)]$ (it is only relative phase delays that are of interest here) we rewrite (1.5-39) as

$$\begin{Bmatrix}V_+(z)\\V_-(z)\end{Bmatrix} = \begin{vmatrix}e^{i\theta_F(z)} & 0\\0 & e^{-i\theta_F(z)}\end{vmatrix}\begin{Bmatrix}V_+(0)\\V_-(0)\end{Bmatrix} \tag{1.5-40}$$

$$\theta_F(z) \equiv \frac{1}{2}(\theta_- - \theta_+) = \frac{\omega}{2c}(n_- - n_+)z \tag{1.5-41}$$

$$\equiv \text{Faraday rotation angle}$$

The reason for calling θ_F the *Faraday rotation angle* becomes clear if we consider the effect of a magnetic medium on an incident wave that is described in the rectangular component representation

$$\begin{pmatrix}V_x(z)\\V_y(z)\end{pmatrix} = T^{-1}\begin{vmatrix}e^{i\theta_F(z)} & 0\\0 & e^{-i\theta_F(z)}\end{vmatrix}T\begin{pmatrix}V_x(0)\\V_y(0)\end{pmatrix}$$

$$= \begin{vmatrix}\cos\theta_F & -\sin\theta_F\\\sin\theta_F & \cos\theta_F\end{vmatrix}\begin{pmatrix}V_x(0)\\V_y(0)\end{pmatrix} \tag{1.5-42}$$

$$= R(-\theta_F)\begin{pmatrix}V_x(0)\\V_y(0)\end{pmatrix} \tag{1.5-43}$$

where $R(-\theta_F)$ is, according to (1.5-2), the matrix representing a *rotation* by $-\theta_F$ about the z axis. The output field is thus rotated by $-\theta_F$ with respect to the input field.

There exists a basic difference between propagation in a magnetic medium and in a dielectric birefringent medium. Consider the latter case first. An x'-polarized eigenwave, for instance, propagating along the z direction in a birefringent crystal has a phase velocity c/n_x, where x' is a principal dielectric axis. The same applies to the wave propagating in the reverse direction. The medium is *reciprocal*. In a magnetic medium the story is quite

[9]The form of T implies that at t=0 the rotating waves $\begin{Bmatrix}1\\0\end{Bmatrix}$ and $\begin{Bmatrix}0\\1\end{Bmatrix}$ are parallel to the x axis.

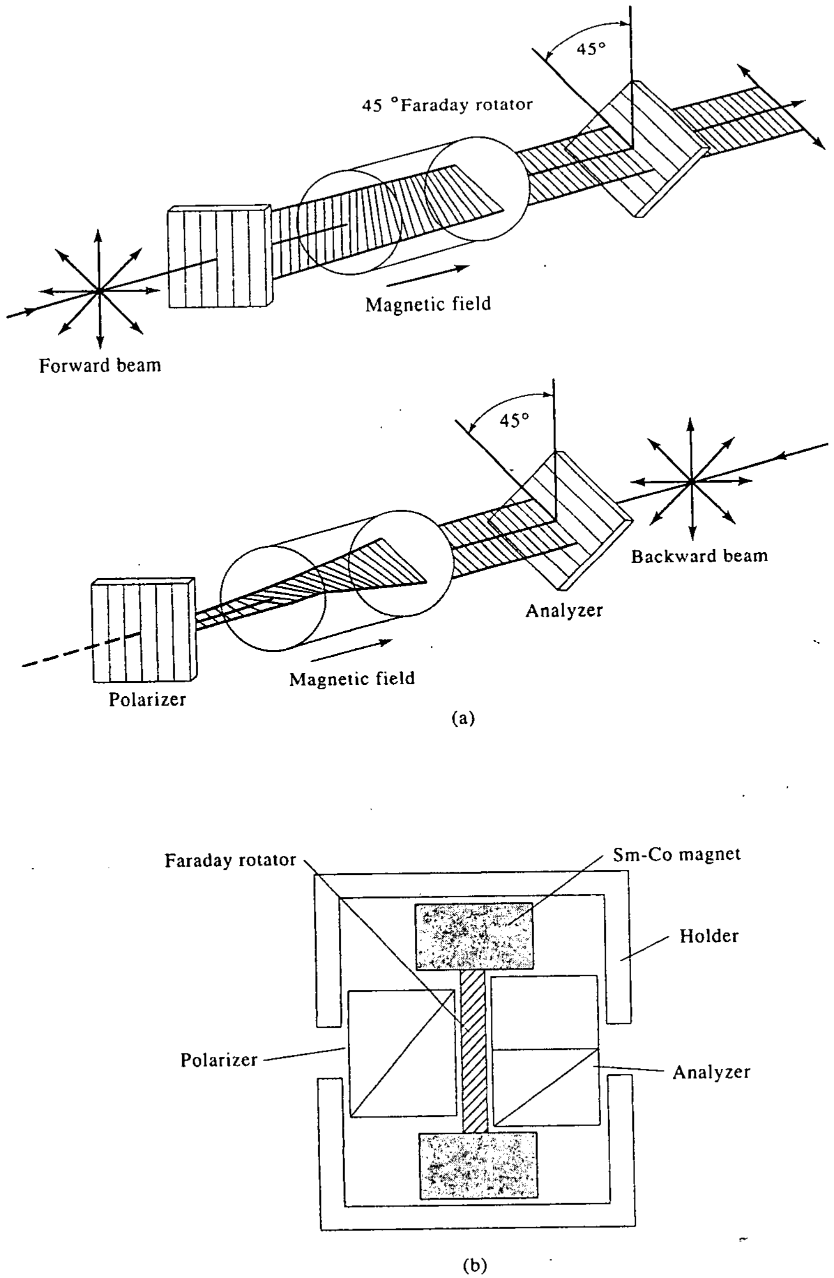


Figure 1-8 (a) A Faraday isolator comprised of two polarizers rotated by 45° relative to each other on either side of a magnetic medium with $\theta_F = 45°$. (b) A cross-sectional view of a practical commercial isolator. (Courtesy of Namiki Precision Jewel Company)

different. Let a linearly polarized wave traveling from left to right a distance L (in the $+z$ direction) undergo a (Faraday) rotation of its plane of polarization of $+\theta$ (the sign signifies the sense of the rotation about the *direction of propagation*). A wave traveling in the $-z$ direction in the crystal will experience a rotation of $-\theta(L)$ *about the new direction* $(-z)$ *of propagation*. This is because the magnetic field or, equivalently, the magnetic polarization now points in the opposite direction relative to the direction of propagation. (The wave can differentiate between $+z$ and $-z$—something that it cannot do in a birefringent crystal). The medium is termed nonreciprocal. The net effect of a round trip through the medium of length L is that the plane of polarization of the beam returning to the starting, $z = 0$ plane, is rotated by $2\theta_F(L)$. This Faraday rotation is used to make optical isolators to block off back-reflected radiation. The basic configuration of a Faraday isolator is illustrated in Figure 1-8 (a) and (b). A linearly polarized incident wave is rotated by 45° in passage through the Faraday medium and then passed fully by the output polarizer. A reflected wave is rotated an additional 45° in the return trip and is thus blocked off by the input polarizer. Faraday isolators now form an integral part of most optical communication systems employing semiconductor diode lasers since such lasers are extremely sensitive to even small amounts of reflected light that cause instabilities in their power and frequency characteristics.

Problems

1.1 Consider the problem of finding the time average

$$\overline{a^2(t)} = \frac{1}{T}\int_0^T a^2(t)\,dt$$

of

$$\begin{aligned} a(t) &= |A_1|\cos(\omega_1 t + \phi_1) + |A_2|\cos(\omega_2 t + \phi_2) \\ &= \mathrm{Re}[V_a(t)] \end{aligned}$$

where

$$V_a(t) = A_1 e^{i\omega_1 t} + A_2 e^{i\omega_2 t}$$

and $A_{1,2} = |A_{1,2}|e^{i\phi_{1,2}}$. $V_a(t)$ is called the *analytical signal* of $a(t)$. Assume that $(\omega_1 - \omega_2) \ll \omega_1$ and integrate over a time T, which is long compared to the period $2\pi/\omega_{1,2}$ but short compared to the beat period $2\pi/(\omega_1 - \omega_2)$.[10] Show that

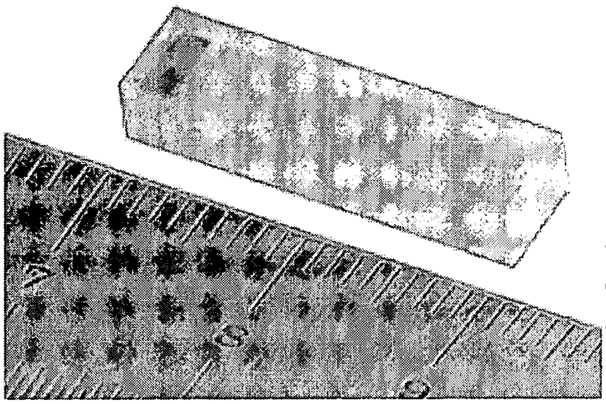
$$\overline{a^2(t)} = \tfrac{1}{2}[V_a(t)V_a^*(t)]$$

[10]When this condition is fulfilled, $a(t)$ consists of a sinusoidal function with a "slowly" varying amplitude and is often called a *quasi-sinusoid*.

Reference 2

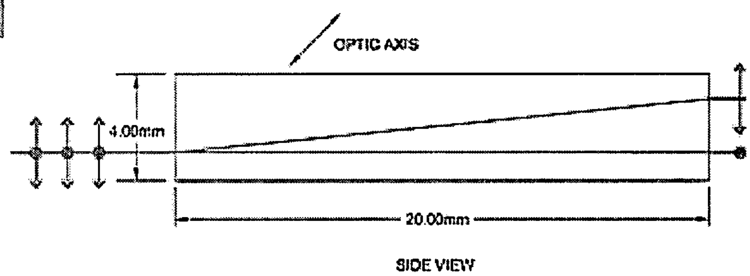
Part Number: **PS802**

Yttrium Vanadate Beam Displacer



- Seperates beam into two polarization dependent parallel beams
- Large output beam seperation - 2.0mm
- Integral part of birefringent optical circulators
- Miniature size - 2.0mm x 4.0mm x 20.0mm
- AR Coated at 1550nm
- Custom sizes available by special order

With a high birefringence, the YVO_4 Beam Displacers offer high seperatioi compact design. The optic axis is oriented at 45° to achieve the beam de· Beam displacers are used in a number of Telecom appications such as op WDMs and polarization beam combiners.



Price: **$150.00** Each, Euro: **€165,00** Availability: **In Stock** Weight: **0.04 lbs, 0.02 kg**
Quantity: 1
ORDER : PS802

2004 Catalog Page
No FAQ's Found

USA
T: 973-579-7227
F: 973-300-3600
sales@thorlabs.com

Germany
T: +49 (0)8131 59560
F: +49 (0)8131 595699
europe@thorlabs.com

Japan
T: +81-3-5977-8401
F: +81-3-5977-8402
sales@thorlabs.jp

Scandinavia
T: +46 31 733 30 00
F: +46 31 703 40 45
scandinavia@thorlabs.com

Unitec
T: +44
F: +44
sales.ul